NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

WARTIME REPORT

NOTES ON THE PROPELLER AND SLIPSTREAM IN

RELATION TO STABILITY

By Herbert S. Ribner

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ADVANCE RESTRICTED REPORT

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· SUMMARY

Charts and formulas are presented in convenient
form for use in the computation of the effects of
propellers on stability. Formulas and curves are given
for

    (a) The propeller "fin effect" in terms of the
        thrust coefficient

    (b) The effect of the wing on the propeller
        normal force in pitch

    (c) The propeller yawing moment due to pitch

    (d) The sidewash induced by the propeller
        in yaw

Formulas are given for the contributions of the direct
propeller forces to the airplane pitching moment, yawing
moment, and shift in neutral point due to power.

INTRODUCTION

In reference 1, an analytical treatment is given of
the effect of propeller operation on the pitching moments
of single-engine monoplanes with flaps neutral. The
analysis, which is based on simplifying assumptions, is
accompanied by experimental data. In reference 2, the
analysis and the experiments are extended to the case
with flaps deflected. The present paper is intended to
supplement and, to a small extent, to amend references 1
and 2. A short extension is added on lateral stability.

Some previous work on propeller "fin effect" (references 3 and 4) has been recast in simpler form with the thrust coefficient replacing the advance-diameter coefficient as the independent variable. The new relation, which is more suitable for stability calculations, is presented herein together with charts for the side-force derivative cross-plotted from reference 3. Hereinafter, the term "normal-force derivative," which is applicable to propellers in both pitch and yaw, is used to replace the term "side-force derivative," which is associated with propellers in yaw.

The upwash or downwash from the wing influences the normal force in pitch. A graph is provided for estimating this change.

The propeller side-force factor was introduced in reference 5 as a criterion for extrapolating side-force charts for one propeller to apply to another propeller. A simpler approximate method that avoids graphical integration is provided herein for readily estimating the side-force factor for a given propeller.

Because the propeller yawing moment due to pitch - or, what is the same except for sign, the pitching moment due to yaw $C_m'_{\psi}$ - has a small but appreciable effect on lateral trim, curves for $C_m'_{\psi}$ have been computed from the formula of reference 3 and are presented herein.

A derivation in reference 4 has been extended to provide a relation for the sidewash of the slipstream of an isolated propeller in yaw that differs from the one in common use (reference 6). A figure is included herein to show the improved agreement with experiment. The new sidewash relation, which is applicable also to the downwash of an isolated propeller in pitch, is presented with graphs to provide ready evaluation of the sidewash or downwash by use of the normal-force graphs of this paper.

SYMBOLS AND DEFINITIONS

The formulas of the present paper, with the exception of those relating to wing upwash, refer to a system of body axes. The origin is at the airplane

center of gravity; the X-axis is parallel to the thrust axis and is directed forward; the Y-axis is directed to the right; and the Z-axis is directed downward. The symbols are defined as follows:

D	propeller diameter
R	tip radius
r	radius to any blade element
S'	disk area $\left(\pi D^2/4\right)$
S	wing area
x	fraction of tip radius (r/R); also distance measured along X-axis
x_0	minimum fraction of tip radius at which shank blade sections develop lift (taken as 0.2)
x_s	ratio of spinner radius to tip radius
b	propeller blade section chord, except in equation (7) where it signifies wing span
$\bar{c}_w$	wing mean aerodynamic chord
z	distance from center of gravity to thrust axis, positive downward
l_1	distance from center of gravity to propeller plane, measured parallel to thrust axis; positive for tractor
V	free-stream velocity
q	free-stream dynamic pressure $\left(\frac{1}{2}\rho V^2\right)$
a	inflow factor $\left(\dfrac{-1 + \sqrt{1 + 8T_c/\pi}}{2}\right)$
n	propeller revolutions per second
β	blade angle to reference chord
β_0	blade angle to zero-lift chord

$\emptyset$ effective helix angle

$$\left[\tan^{-1}\left(\frac{V(1 + a)}{2\pi r - \text{Slipstream rotational velocity}}\right)\right]$$

ψ angle of yaw, radians

α angle of attack

ϵ angle of downwash

ϵ_1 value of wing downwash for $C_{L_w} = 1$, (ϵ/C_L)

ϵ_o downwash at $\alpha_T = 0^o$

σ angle of sidewash

C_L lift coefficient

C_{L_α} lift-curve slope, per radian $\left(\frac{dC_L}{d\alpha}\right)$

m_o section lift-curve slope, per radian

C_{Y}' propeller side-force coefficient (Side force/qS')

$C_{Y}'_\psi$ derivative of propeller side-force coefficient
         with respect to yaw $(\partial C_{Y}'/\partial \psi)$; more
         generally, derivative of propeller normal-
         force coefficient with respect to angle of
         inclination of thrust axis in radians
         (coefficient based on disk area)

$C_m'_\psi$ derivative of propeller pitching-moment coef-
         ficient with respect to yaw $\left[\frac{\partial}{\partial \psi}\left(\frac{\text{Pitching moment}}{qDS'}\right)\right]$

$C_{Y}'_{\psi_o}$ value of $C_{Y}'_\psi$ for $T_c = 0$

T_c thrust coefficient (Thrust/$\rho V^2 D^2$)

f ratio of $C_{Y}'_\psi$ to $C_{Y}'_{\psi_o}$; called f(a) in
         references 3 and 4

f_m ratio of $C_m'_\psi$ to $C_m'_{\psi_o}$

S.F.F.   abbreviation for side-force factor, defined in
          equation (3)

C_m airplane pitching-moment coefficient
          (Pitching moment/$qS\bar{c}_w$)

C_n airplane yawing-moment coefficient
          (Yawing moment/qSb)

ΔC_{m_P} increment in C_m due to direct forces on
          propeller

ΔC_{n_P} increment in C_n due to direct forces on
          propeller

Q_c propeller torque coefficient (Torque/$\rho V^2 D^3$)

Subscripts:

T   thrust axis

w   wing

o   at zero thrust when applied to force and moment
          coefficients

c   corrected for compressibility

Definitions:

Stick-fixed neutral point: center-of-gravity location
    for zero stick travel with change of trim speed for
    straight flight at constant throttle; distance of
    stick-fixed neutral point forward of origin of moments
    is given by

$$\frac{x}{\bar{c}_w} = \left[\left(\frac{\partial C_m}{\partial C_L}\right)_{T_c} + \left(\frac{\partial C_m}{\partial T_c}\right)_\alpha \frac{dT_c}{dC_L}\right]_{\text{elevator fixed}}$$

Stick-free neutral point: center-of-gravity location for
    no change from zero stick force with change of trim
    speed for straight flight at constant throttle;
    distance of stick-free neutral point forward of origin
    of moments is given by

$$\frac{x}{\bar{c}_w} = \left[\left(\frac{\partial C_m}{\partial C_L}\right)_{T_c} + \left(\frac{\partial C_m}{\partial T_c}\right)_\alpha \frac{dT_c}{dC_L}\right]_{\text{elevator free}}$$

Aerodynamic center: center-of-gravity location for zero
   change in pitching moment with lift coefficient at
   constant throttle when airplane is held in air stream
   at constant speed and angle of attack is varied;
   distance of aerodynamic center forward of origin of
   moments is given by

$$\frac{x}{c_w} = \left(\frac{\partial C_m}{\partial C_L}\right)_{T_c}$$

(Determined for straight flight. Aerodynamic center
may be called stick fixed or stick free depending on
whether pitching moments are determined with elevator
fixed or free.)

Maneuver point: center-of-gravity location for zero
   change of stick force with acceleration in maneuvers
   at constant speed; distance of maneuver point forward
   of origin of moments is given by

$$\frac{x}{c_w} = \left(\frac{\partial C_m}{\partial C_L}\right)_{T_c}$$

(Determined for curved flight with elevator free.)

PROPELLER NORMAL FORCE

Isolated Propeller

     The thrust coefficient T_c has emerged as the
natural independent variable for the presentation of the
effects of power on stability and control. It has
therefore appeared desirable to recast the formulas of
references 3 and 4 on propeller fin effect with the
thrust coefficient replacing the advance-diameter coef-
ficient as the independent variable; moreover, a certain
approximation is thereby suggested that provides a
considerable simplification of the result. Thus it has
been observed that the expression multiplying f(a) in
equations (1) and (2) of reference 3 is essentially
constant; the variation is in almost all cases within
±4 percent. Defining the value of this expression

for $T_c = 0$ as $C_{Y'_{\psi_0}}$, calling $f(a)$ shortly f, and noting that $f = 1$ for $T_c = 0$ results in

$$C_{Y'_\psi} = \frac{\partial Y/\partial \psi}{qS'} = fC_{Y'_{\psi_0}} \tag{1}$$

where f depends solely on the thrust coefficient T_c, and $C_{Y'_{\psi_0}}$ depends solely on the propeller geometry and whether single or dual rotation is provided. The quantity $f - 1$ is about two-thirds the ratio of the dynamic-pressure increment at the propeller disk to the dynamic pressure in the free stream. The factor $C_{Y'_{\psi_0}}$ is essentially proportional to the projected side area of the propeller - that is, one-half the number of blades times the area that a blade appears to have when viewed from the side.

Formulas for f and $C_{Y'_{\psi_0}}$ are given in the appendix. The variation of f with T_c, which is independent of the propeller used, is presented in figure 1. The variation of $C_{Y'_{\psi_0}}$ with blade angle and with number of blades for single and for dual rotation is presented in figure 2 for the Hamilton Standard 3155-6 propeller and in figure 3 for the NACA 10-3062-045 propeller. The curves of these two figures were obtained by cross-plotting the line of zero thrust against the blade angle from the curves of reference 3. The geometric characteristics of the two propellers are shown in figure 4, which was taken from reference 3.

Correction for Wing Upwash

The normal-force charts (figs. 1 to 3) include a correction (spinner factor) for spinner-nacelle interference. In addition, however, the upwash or downwash from the wing alters the propeller normal force in pitch in the ratio $\left(1 - \frac{d\epsilon}{d\alpha}\right)$ to 1. This ratio is given in figure 5 for elliptic wings of aspect ratios 6, 9, and 12 as a function of the longitudinal

distance from the root quarter-chord point. The values
ahead of the wing are taken along the line through the
wing-root leading edge parallel to the free stream, and
the values behind the wing are taken along the inclined
trailing vortex sheet. (See reference 7.) The compu-
tations are based on a wing representation of four
straight lifting lines arranged to approximate stepwise
the theoretical spanwise and chordwise loading. The
section lift-curve slope of 2π was chosen in order to
provide some allowance for the increase in upwash due to
the augmented lift of the wing in the presence of the
slipstream.

The values given in figure 5 cover the region close
to the wing. For larger distances (>0.4 semispan)
from an origin in the wing, the extensive downwash charts
of reference 7, computed for the region behind the wing,
may be applied to determine ϵ in front of the wing by
means of the relation

$$\epsilon(-x,z) = C_{L_w}\left[\epsilon_1(x,z) - \epsilon_1(\infty,z)\right] \qquad (2)$$

The notation $\epsilon(x,z)$ signifies the value of ϵ at the
point (x,z) relative to wing root quarter-chord point,
with x measured downstream and z measured upward
perpendicular to x in the plane of symmetry. The
value ϵ_1 is the charted value of ϵ in reference 7
for $C_{L_w} = 1$. The accuracy of this method is limited by
the necessity for extrapolating the charted downwash
contours to determine $\epsilon_1(\infty,z)$. The origin of coordinates
should be shifted to the 0.5-chord point when the upwash
due to flap deflection is estimated.

Side-Force Factor

For convenience, the self-explanatory figures 6 and 7
are reproduced from reference 5 to provide conversion
factors for applying the curves of figures 2 and 3 to any
specific propeller in terms of the side-force factor for
that propeller. The side-force factor, which is a
measure of the relative effectiveness of a propeller
blade in developing side force, is defined in reference 5
by the formula

$$\text{S.F.F.} = \frac{(10)^5}{32} \int_{0.2}^{1.0} \frac{b}{D} \sin\left(\beta - \beta_{0.75R} + 25°\right) d\frac{r}{R} \qquad (3)$$

The evaluation of the sine term is somewhat laborious and yet the integral is not sensitive to the variations in this term that occur among different propellers. This complication and the usual graphical integration are avoided by the use of an average curve of β against $\frac{r}{R}$ in a formula for approximate integration. There is obtained the approximation

$$S.F.F. = 525\left(\frac{b}{D}\right)_{0.3} + 525\left(\frac{b}{D}\right)_{0.6} + 270\left(\frac{b}{D}\right)_{0.9} \qquad (4)$$

where the subscripts designate the abscissas of the curve of b/D against r/R at which b/D is to be measured. Equation (4) is accurate to about ± 4 percent.

The change of the basis of comparison from constant V/nD in reference 5 to constant T_c herein is quantitatively unimportant.

It should be remarked that it would not be profitable to complicate the definition for side-force factor (equation (3)) to account for variations in spinner radius. The augmentative effect of the spinner (see reference 4, page 28) increases with spinner radius by an amount roughly sufficient to offset the loss of lift from the blade sections covered by the spinner.

Correction for Compressibility

No experimental data appear to be available on the effect of compressibility on propeller normal force. From theoretical considerations, it is shown in appendix B of reference 4 that a first-order correction for compressibility is obtained by dividing the normal force by $\sqrt{1 - M_e^2}$, where M_e is related to the stream Mach number M and to V/nD by the curve reproduced for convenience herein as figure 8. The correction is invalid above the critical Mach number for the propeller.

PITCHING MOMENT FROM DIRECT PROPELLER FORCES

If the foregoing formulas for propeller normal force are introduced into reference 1, formula (7) therein for

the contribution of the direct propeller forces to the
airplane pitching moment assumes the form

$$\Delta C_{m_P} = \frac{S'}{S} \left\{ \frac{8}{\pi} \frac{z}{\overline{c}_w} T_c + fC_{Y'}_{\psi_o} \left[\left(1 - \frac{d\epsilon}{d\alpha} \right) \alpha_T - \epsilon_o \right] \frac{l_1}{\overline{c}_w} \right\} \quad (5)$$

where the symbol S' is the propeller-disk area and $-\epsilon_o$
is the upwash at $\alpha_T = 0^o$ due to flap deflection and wing
incidence. The factors of the second term within the
braces may be evaluated from the procedures of this
report.

The contribution of the direct propeller forces to
the forward shift due to power of either the stick-fixed
or the stick-free neutral point is obtained from
formula (5) as

$$\frac{\Delta x}{\overline{c}_w} = \frac{d\Delta C_{m_P}}{dC_L} = \frac{S'}{S} \left[\frac{8}{\pi} \frac{z}{\overline{c}_w} \frac{dT_c}{dC_L} + fC_{Y'}_{\psi_o} \left(1 - \frac{d\epsilon}{d\alpha} \right) \frac{l_1}{\overline{c}_w} \frac{1}{C_{L_\alpha}} \right] \quad (6)$$

where C_{L_α} is the power-on lift-curve slope per radian.
A small term involving $d\left(fC_{Y'}_{\psi_o} \right) \big/ dC_L$ has been omitted.

The corresponding forward shift of the aerodynamic center
is obtained by omitting the dT_c/dC_L term of equation (6)
(a small term is again neglected). The revised expression
gives also the approximate forward shift of the maneuver
point, since the direct propeller forces do not appre-
ciably affect the separation of the maneuver point and
the aerodynamic center. The terms "neutral point" and
"maneuver point" have not come into universal usage. These
two terms and, for comparison, the "aerodynamic center"
are therefore formally defined under "Symbols and
Definitions."

It is emphasized that formulas (5) and (6) and the
attendant remarks on the shifts of the neutral point,
aerodynamic center, and maneuver point apply only to
the effects of the direct propeller forces. The effects
of the propeller slipstream, which for the neutral point
and aerodynamic center are of comparable or greater
magnitude, are treated in references 1 and 2.

YAWING MOMENT FROM DIRECT PROPELLER FORCES

For single-engine airplanes and for multiengine
airplanes under symmetric power conditions, the contribu-
tion of the direct propeller forces to the yawing-moment
coefficient of the airplane may be written (with ψ in
radians and N the number of propellers)

$$\Delta C_{n_p} = N \frac{S'}{S} \left\{ f C_{Y'}{}_{\psi_0} \frac{l_1}{b}\psi - C_{m'}{}_{\psi}\left[\left(1 - \frac{d\epsilon}{d\alpha}\right)\alpha_T - \epsilon_0\right]\frac{D}{b}\right\} \qquad (7)$$

with respect to axes defined in the section "Symbols and
Definitions." The $C_{Y'}{}_{\psi_0}$ term represents the propeller
fin effect already discussed. The $C_{m'}{}_{\psi}$ term represents
a yawing moment at zero yaw due to the asymmetric thrust
loading of a single-rotating propeller in pitch. (The
moment vanishes with dual rotation.) The computed
magnitude of the $C_{m'}{}_{\psi}$ term amounts to about one-third
the magnitude observed in unpublished tests at LMAL of
the left yawing moment at zero yaw for the P-47B airplane
with tail off, flaps deflected 40°, $\alpha_T = 10.2°$,
$T_c = 0.37$, and $Q_c = 0.045$. Adding the tail more than
doubles the observed yawing moment and reduces the
computed direct propeller contribution to one-seventh
of the total. The interference of the slipstream with
the wing-fuselage combination and the tail, which accounts
for the remaining six-sevenths, cannot be treated here.

The equation for $C_{m'}{}_{\psi}$ is given in references 3
and 4, together with aids for its evaluation. Because
$C_{m'}{}_{\psi}$ has the previously described small effect on
lateral trim, the computations have been carried out
for the three- and four-blade Hamilton Standard pro-
pellers 3155-6. The results are presented herein in
figures 9 to 11 in terms of thrust coefficient, blade
angle, and number of blades. It is suggested that the
side-force factor (see section "Side-Force Factor") be
used in case it is necessary to obtain approximate
values for other propellers. Several values of $C_{m'}{}_{\psi}$
extrapolated in this way to apply to the
NACA 10-306-045 propeller were found to agree with direct
computed values within ±10 percent.

SIDEWASH IN SLIPSTREAM

The sidewash in the slipstream of an isolated yawed propeller may be obtained from equation (12) of reference 4. The quantity v_y therein is the sidewash velocity far back. Inserting

$$V_a = V(1 + a)$$

$$\frac{2}{\pi}T_c = a(1 + a)$$

and

$$\frac{2}{\pi}Y_c = \frac{C_Y'}{4}$$

permits solving for the sidewash derivative in the form

$$\frac{d\sigma}{d\psi} = \frac{d}{d\psi}\left[\frac{v_y}{V(1 + 2a)}\right]$$

$$= A + BC_Y'_\psi \tag{8}$$

where

$$A = \frac{2a(1 + 2a)}{1 + (1 + 2a)^2}$$

$$B = \frac{1 + 2a}{2(1 + 2a)\left[1 + (1 + 2a)^2\right]}$$

Since, by equation (1),

$$C_Y'_\psi = fC_Y'_{\psi_o}$$

a convenient alternative form is

$$\frac{d\sigma}{d\psi} = A + B'C_Y'_{\psi_o} \tag{9}$$

where $B' = fB$. The quantities A, B, and B' are the functions of T_c given in figure 12.

Equations (8) and (9) are applicable to the downwash of an isolated propeller in pitch if $\frac{d\epsilon}{d\alpha}$ is written for $\frac{d\sigma}{d\psi}$. With this change, these equations may be used to replace equation (15) of reference 1, and figures 2, 3, and 12 herein may be used in place of figures 4 and 31 of that report.

The new formula (equation (8)) differs from the formulas of Glauert (reference 6) in that the reaction of the air displaced by the sidewash of the slipstream is taken into account. The several formulas are compared with experimental data of reference 8 in figure 13. The experimental data were taken at a station slightly more than 1 diameter behind the propeller; therefore, the curve labeled "at disk" is theoretically not applicable. (Note that 1 diameter may be considered "far" behind the propeller as regards the axial slipstream velocity, since 95 percent of the final inflow velocity is attained at this distance.) As distinguished from the "in wake" curve, however, the "at disk" curve agrees fairly well with the experimental points, and the corresponding equation appears to be generally applied in the literature to the downwash far back. The new formula (equation (8)) shows better agreement with experiment and is not subject to the technical objection of misapplication.

RESUME

1. The expression for propeller normal force (fin effect) may be simplified into a product of a function of the thrust coefficient and a function of the blade geometry that is essentially the projected side area of the propeller. A few curves serve for an entire family of propellers and blade angles.

2. Procedures are given for estimating the upwash or downwash from the wing, which influences the propeller normal force in pitch.

3. An approximate method is provided for readily estimating the side-force factor for a given propeller from its plan-form curve without graphical integration.

4. An improved relation is provided for the side-
wash in the slipstream of an isolated propeller in yaw
in terms of the thrust coefficient and the projected
side area of the propeller.

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APPENDIX

FORMULAS FOR f AND $c_{Y'_{\psi_o}}$

Formulas for f and $c_{Y'_{\psi_o}}$, derived from reference 3, are as follows:

$$f = \frac{(1 + a)\left[(1 + a) + (1 + 2a)^2\right]}{1 + (1 + 2a)^2} \tag{12}$$

For single rotation,

$$c_{Y'_{\psi_o}} = \frac{k_s\left(\sigma I_1 - \dfrac{\sigma^2 I_2{}^2}{1 + \sigma I_3}\right)}{1 + k_{a_o}\left(\sigma I_1 - \dfrac{\sigma^2 I_2{}^2}{1 + \sigma I_3}\right)} \tag{13}$$

For dual rotation, the second term in the parentheses both of the numerator and the denominator of equation (13) is omitted and

$$c_{Y'_{\psi_o}} = \frac{k_s \sigma I_1}{1 + k_{a_o} \sigma I_1}$$

and for both cases,

$$k_s = 1 + \frac{K\displaystyle\int_{x_o}^{1} \left(\frac{x_s}{x}\right)^2 \frac{b}{b_{0.75R}} \sin\beta_o \, dx}{\displaystyle\int_{x_o}^{1} \frac{b}{b_{0.75R}} \sin\beta_o \, dx}$$

$$k_{a_o} = \frac{1}{8} \frac{\displaystyle\int_{x_o}^{1} \frac{b^2}{b_{0.75R}{}^2} \frac{\sin^2\beta_o \, dx}{x}}{\left(\displaystyle\int_{x_o}^{1} \frac{b}{b_{0.75R}} \sin\beta_o \, dx\right)^2}$$

$$\sigma = \frac{4}{3\pi} \left(\frac{b}{D}\right)_{0.75R} \quad \text{times number of blades}$$

$$I_1 = \frac{3}{4} m_o \int_{x_o}^{1} (b/b_{0.75R}) \sin \beta_o \, dx$$

$$I_2 = \frac{3}{4} m_o \int_{x_o}^{1} (b/b_{0.75R}) \cos \beta_o \, x \, dx$$

$$I_3 = \frac{3}{4} m_o \int_{x_o}^{1} \frac{b}{b_{0.75R}} \frac{\cos^2 \phi}{\sin \phi} \, x^2 \, dx$$

and K varies from a value of 0.9 for a nacelle fineness ratio of 6 to a value of 1.00 for a fineness ratio of infinity. For a discussion of the factors involved and a chart for the determination of I_3, see references 3 and 4.

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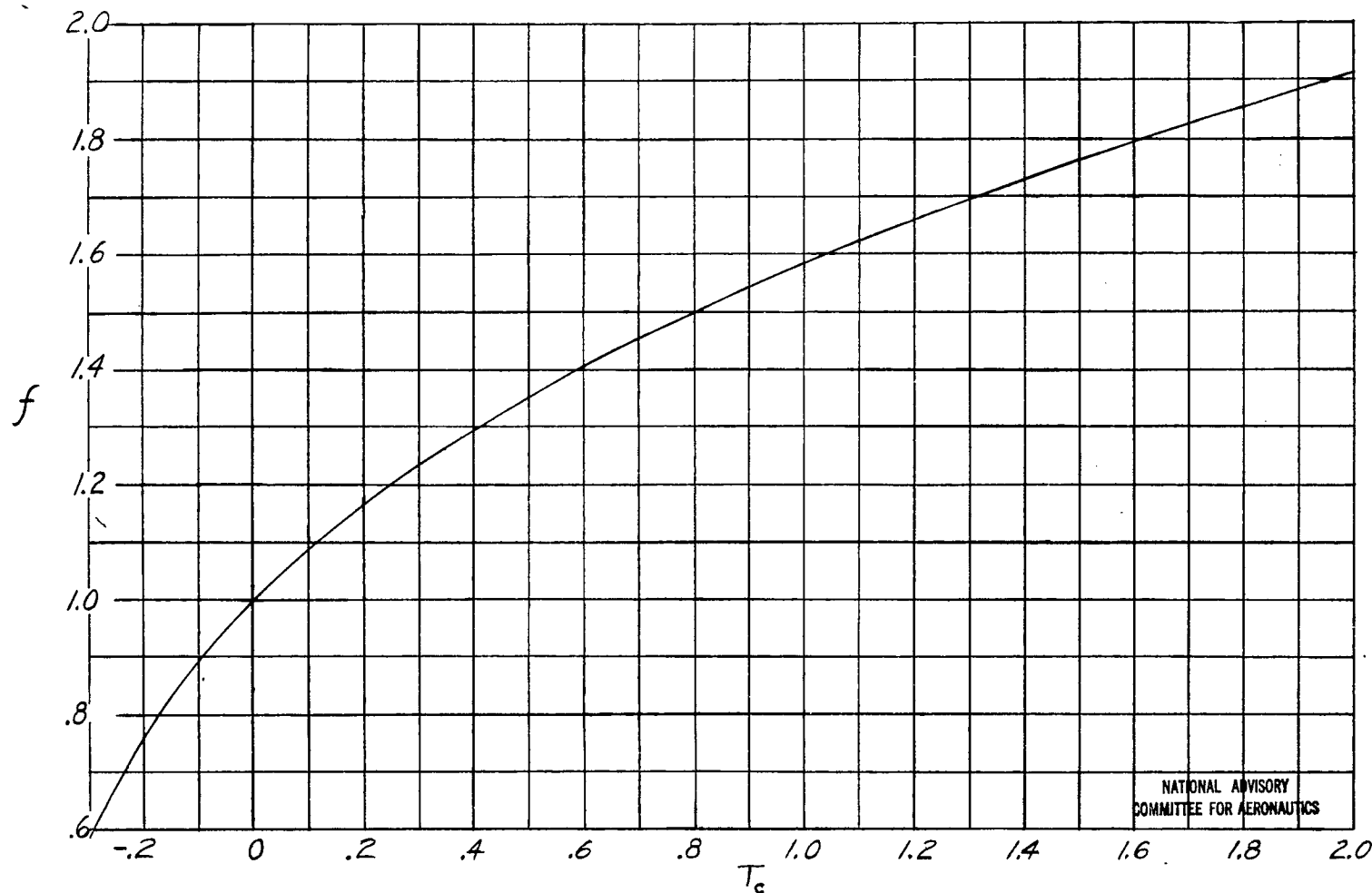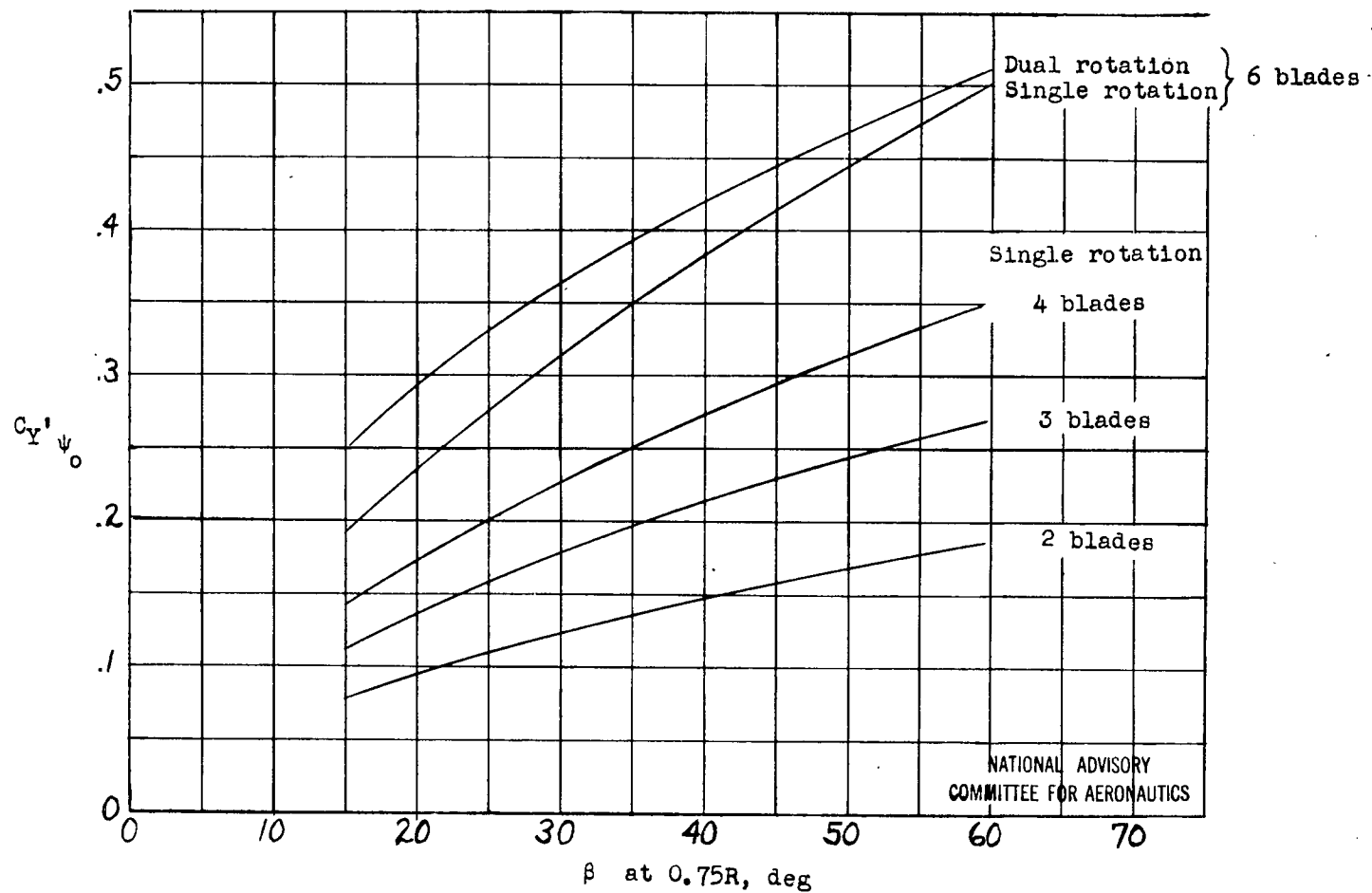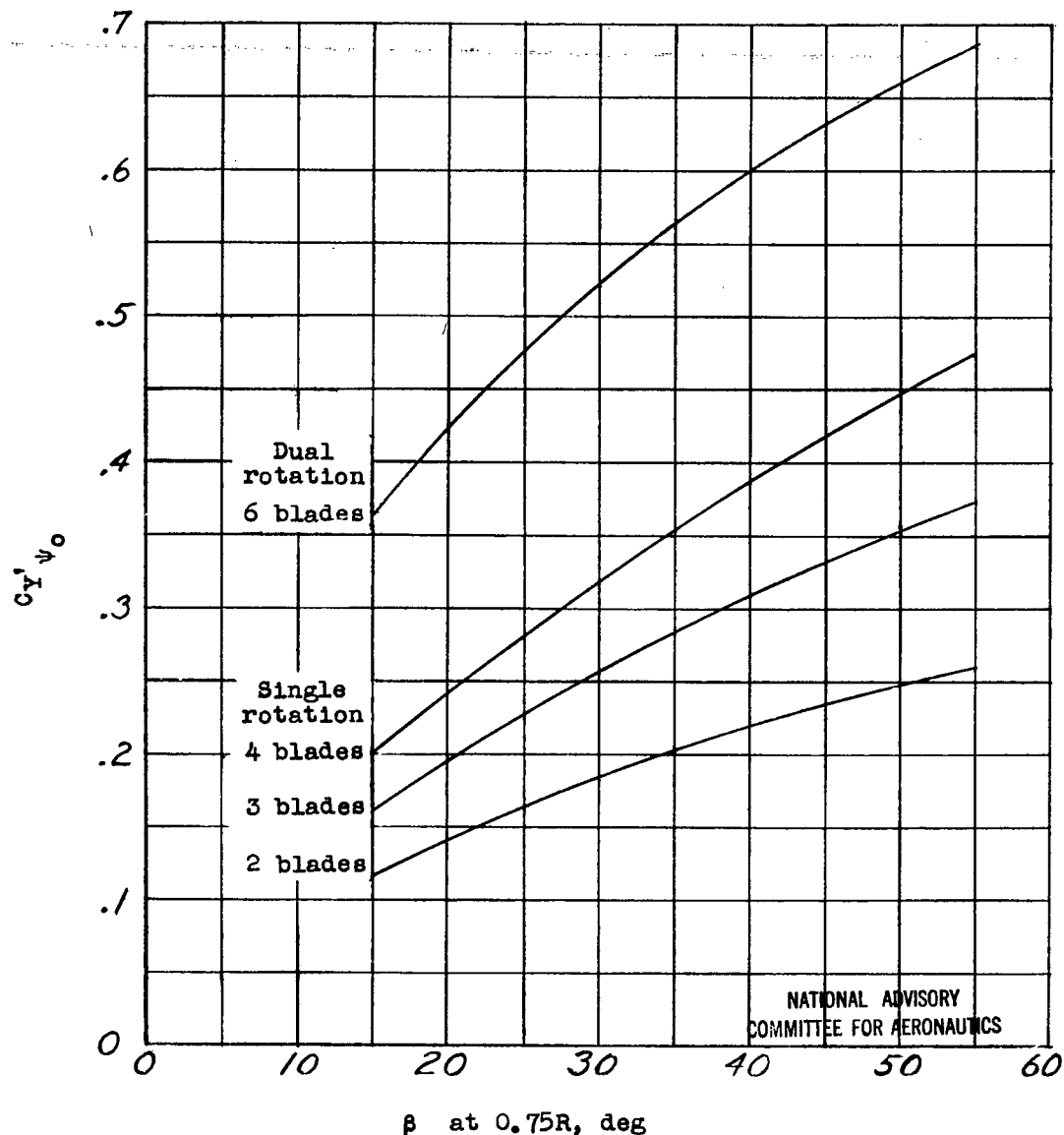
Figure 1.- Variation of f with T_c for use in formula $\dfrac{\text{Propeller normal force}}{\frac{1}{2}\rho V^2 \times \text{Wing area}} = \dfrac{\text{Disk area}}{\text{Wing area}} f C_{Y'}{}_{\psi_0} \psi (\text{radians})$.

Product $f C_{Y'}{}_{\psi_0}$ is designated $C_{Y'}{}_\psi$. Factor f is same for all propellers.

Fig. 1

Fig. 2

Figure 2.- Variation of $C_{Y'}{}_{\psi_o}$ with blade angle for use in formula

$$\frac{\text{Propeller normal force}}{\frac{1}{2}\rho V^2 \times \text{Wing area}} = \frac{\text{Disk area}}{\text{Wing area}} f C_{Y'}{}_{\psi_o} \psi \text{ (radians)}. \quad \text{Product} \quad f C_{Y'}{}_{\psi_o}$$

is designated $C_{Y'}{}_{\psi}$. Hamilton Standard 3155-6 propeller with 0.164-diameter spinner; x_o taken as 0.2; S.F.F. = 80.7.

Figure 3.— Variation of $C_Y'{}_{\psi_o}$ with blade angle for use in formula

$$\frac{\text{Propeller normal force}}{\tfrac{1}{2}\rho V^2 \times \text{Wing area}} = \frac{\text{Disk area}}{\text{Wing area}} f C_Y' \ \psi \ (\text{radians}).\quad \text{Product} \ f C_Y'{}_{\psi_o}$$

is designated $C_Y'{}_\psi$. NACA propeller 10-3062-045 with 0.164-diameter spinner; x_o taken as 0.2; S.F.F. = 131.6.
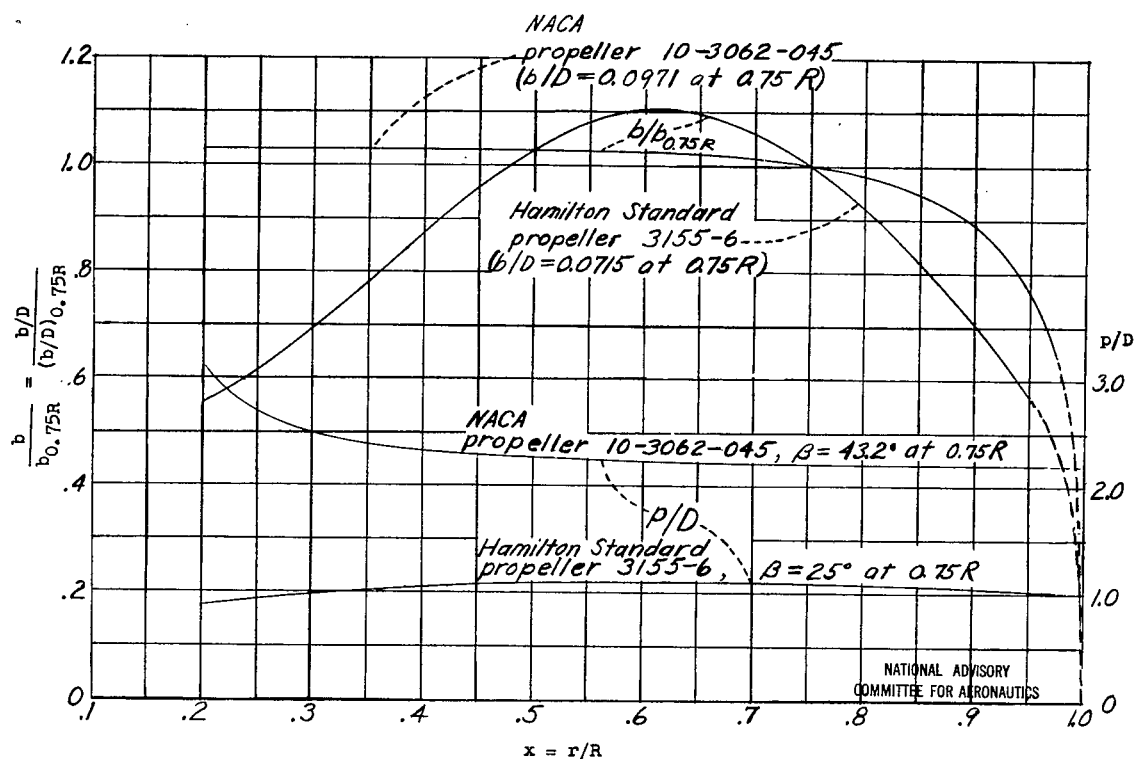
Figure 4.- Plan-form curves and pitch distributions of NACA 10-3062-045
and Hamilton Standard 3155-6 propellers.
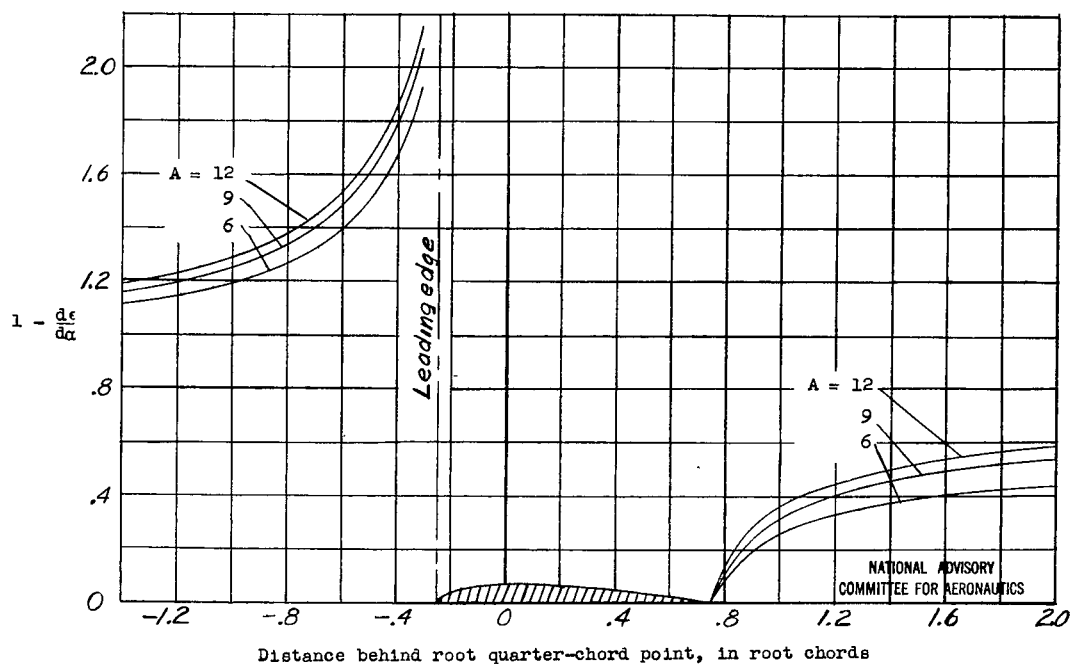
Figure 5.- Value of $1 - \dfrac{d\epsilon}{d\alpha}$ on longitudinal axis of elliptic wing
for aspect ratios 6, 9, and 12.
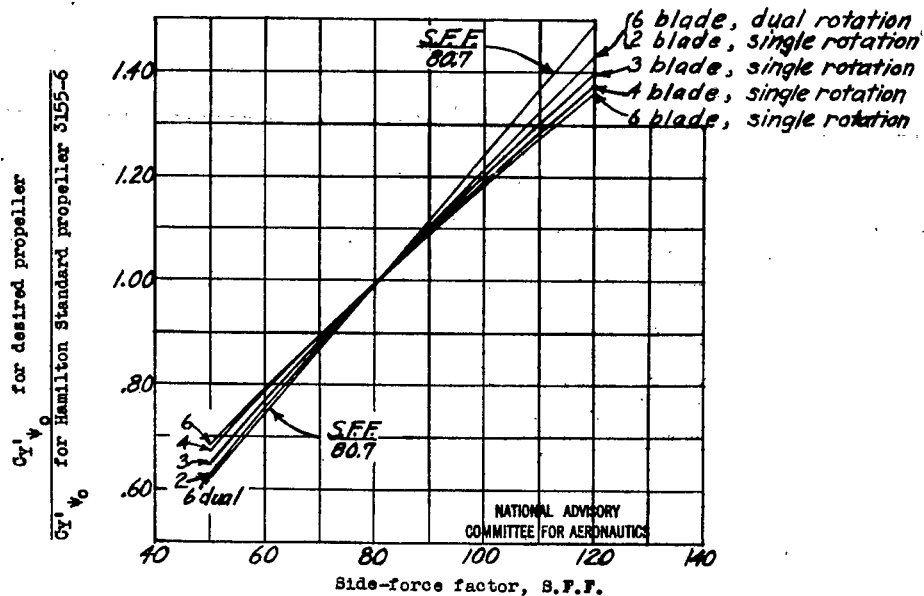
Figure 6.- Ratio of side-force derivatives $\dfrac{C_{Y'_{\psi_0}} \text{ for desired propeller}}{C_{Y'_{\psi_0}} \text{ for Hamilton Standard propeller 3155-6}}$

as a function of side-force factor of desired propeller. For extrapolation from side-force charts for Hamilton Standard propeller 3155-6, for which side-force factor is 80.7. Approximate ratio S.F.F./80.7 is included for comparison. (From reference 5.)
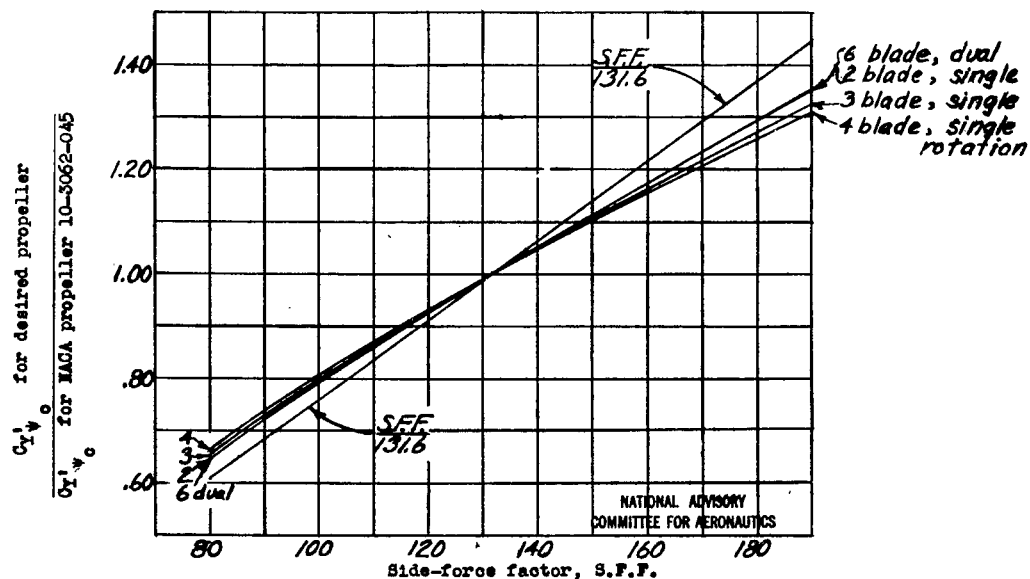

Figure 7.- Ratio of side-force derivatives $\dfrac{C_{Y'_{\psi_0}} \text{ for desired propeller}}{C_{Y'_{\psi_0}} \text{ for NACA propeller 10-3062-045}}$

as a function of side-force factor of desired propeller. For extrapolation from side-force charts for NACA propeller 10-3062-045, for which side-force factor is 131.6. Approximate ratio S.F.F./131.6 is included for comparison. (From reference 5.)
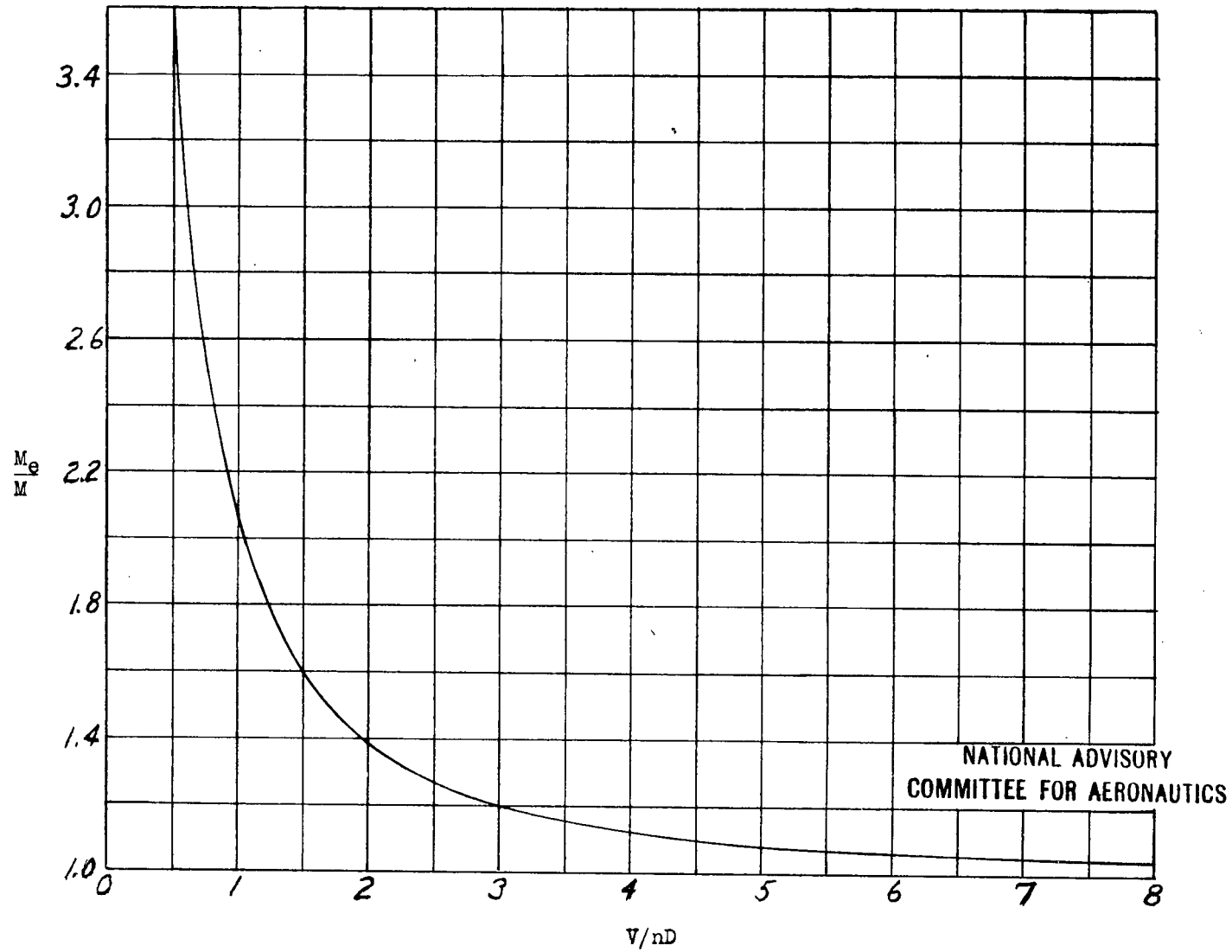
Fig. 8

Figure 8.- Variation of ratio (Effective Mach number)/(Stream Mach number) with V/nD for use in relation $C_{Y'_{\psi_c}} = C_{Y'_\psi} / \sqrt{1 - M_e^2}$.
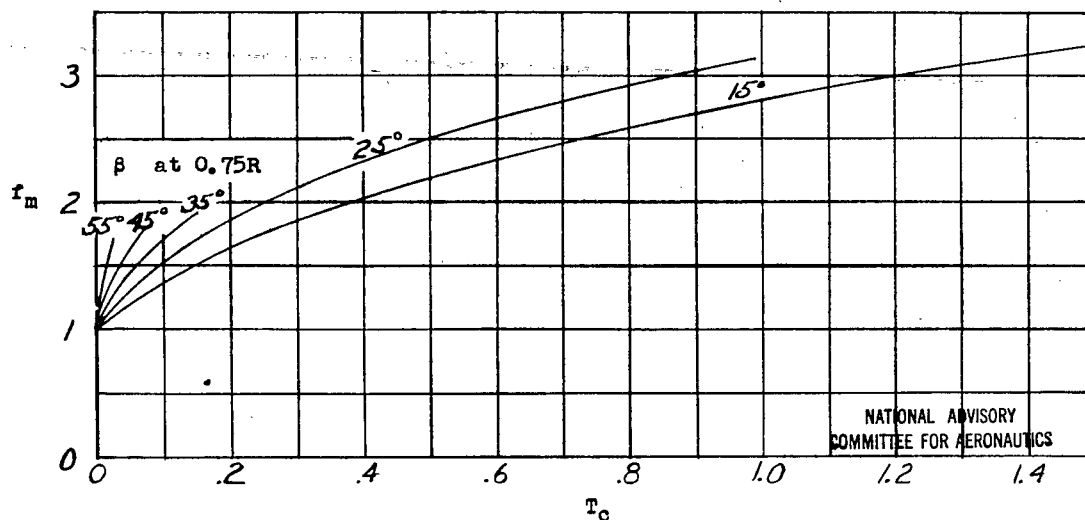
Figure 9.– Variation of f_m with T_c for use in formula $C_m{}'_\psi = f_m C_m{}'_{\psi_0}$.
Hamilton Standard propeller 3155-6 with 0.164-diameter spinner; x_0 taken as 0.2; S.F.F. = 80.7; three blades.
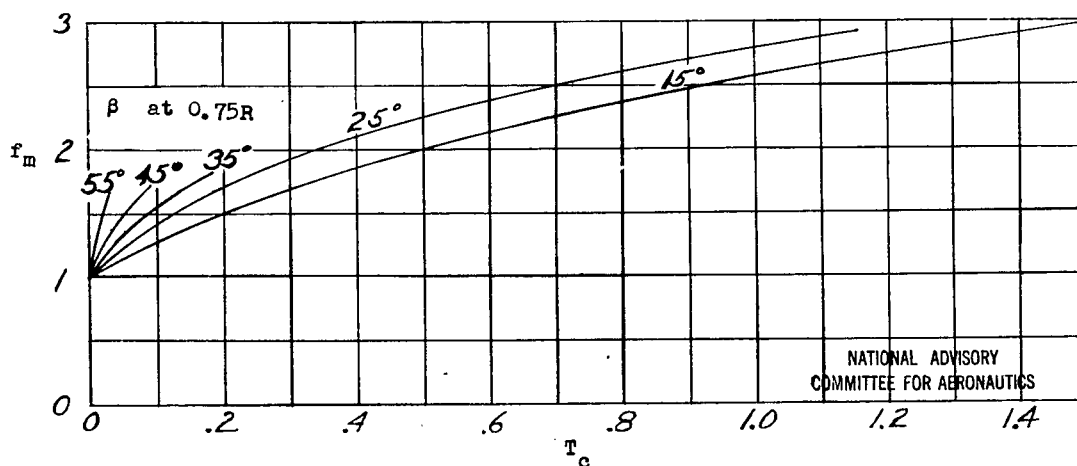

Figure 10.– Variation of f_m with T_c for use in formula $C_m{}'_\psi = f_m C_m{}'_{\psi_0}$.
Hamilton Standard propeller 3155-6 with 0.164-diameter spinner; x_0 taken as 0.2; S.F.F. = 80.7; four blades.